

$$: \langle \psi_{l,m+2} | L_x^2 | \psi_{l,m} \rangle.$$

$$= \langle \psi_{l,m+2} | \left(\frac{L_+ + L_-}{2} \right) \left(\frac{L_+ + L_-}{2} \right) | \psi_{l,m} \rangle.$$

$$\left(\begin{array}{l} L_+ \& L_- \\ \text{are ladder} \\ \text{operators} \end{array} \right) \quad \left(\begin{array}{l} \text{where} \\ L_+ = L_x + iL_y \\ L_- = L_x - iL_y \end{array} \right)$$

$$= \frac{1}{4} \langle \psi_{l,m+2} | L_+^2 + L_+ L_- + L_- L_+ + L_-^2 | \psi_{l,m} \rangle.$$

$$= \frac{1}{4} \left(\langle \psi_{l,m+2} | L_+^2 | \psi_{l,m} \rangle + \langle \psi_{l,m+2} | L_+ L_- | \psi_{l,m} \rangle \right. \\ \left. + \langle \psi_{l,m+2} | L_- L_+ | \psi_{l,m} \rangle + \langle \psi_{l,m+2} | L_-^2 | \psi_{l,m} \rangle \right)$$

$$= \frac{1}{4} \left(A \langle \psi_{l,m+2} | \psi_{l,m+2} \rangle + A \langle \psi_{l,m+2} | \psi_{l,m-2} \rangle \right)$$

combine $\rightarrow 0$

\therefore orthogonal

$$+ \langle \psi_{l,m+2} | L^2 - L_z^2 + \frac{1}{2} L_+ L_- + L^2 - L_z^2 - \frac{1}{2} L_- L_+ | \psi_{l,m} \rangle$$

$$\frac{1}{4} \left(A \langle \psi_{l,m+2} | \psi_{l,m+2} \rangle + 2 \langle \psi_{l,m+2} | L^2 | \psi_{l,m} \rangle \right)$$

$$= 2 \langle \psi_{l,m+2} | L^2 | \psi_{l,m} \rangle$$

$$= \cancel{\frac{\phi}{4}}$$

$$= \frac{1}{4} (A + 2B' \langle \cancel{\psi_{l,m+2}} | \cancel{\psi_{l,m}} \rangle - C' \langle \cancel{\psi_{l,m+2}} | \cancel{\psi_{l,m}} \rangle)$$

orthogonal.

$$= \frac{1}{4} A$$

$$= \frac{1}{4} \hbar^2 \sqrt{l(l+1) - m(m+1)} \sqrt{l(l+1) - (m+1)(m+2)}$$

$$\left(L_+ | \psi_{l,m} \rangle = \hbar \underbrace{\sqrt{l(l+1) - m(m+1)}}_{\downarrow A'} | \psi_{l,m+1} \rangle \right)$$

$$\left(L_+ | A' \psi_{l,m+1} \rangle = A' \hbar \sqrt{l(l+1) - (m+1)(m+2)} | \psi_{l,m+2} \rangle \right)$$