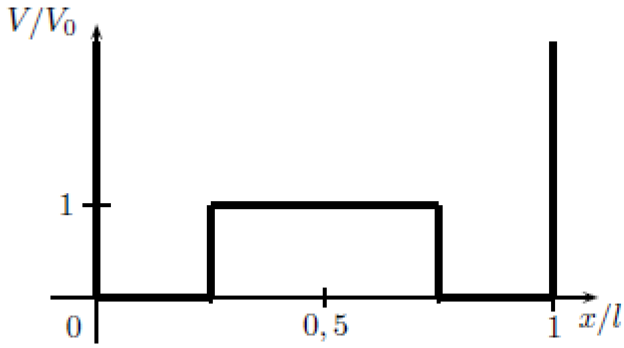
Variational Principle

Consider a one-dimensional system with a particle located in

the following potential:



$$V = \begin{cases} V_0 & \text{for } \frac{1}{4}l \leq x \leq \frac{3}{4}l \\ 0 & \text{for } 0 \leq x < \frac{1}{4}l \text{ and } \frac{3}{4}l < x \leq l \\ \infty & \text{otherwise} \end{cases}$$

The Hamiltonian operator for this system is:

$$\hat{H} = \hat{T} + \hat{V} = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V$$

where m is the mass of the particle.

A) First show for $a \in \mathbb{R}$ by explicit integration that:

$$\int \sin^2(ax)\mathrm{d}x = \frac{1}{2}x - \frac{1}{2a}\sin(ax)\cos(ax)$$

B) The test wave function is given: $\phi_1 = \sin\left(\pi l^{-1} x\right).$

Check if the wave function is normalized. If necessary, determine the normalization factor. Then give the normalized function.

C) Determine the expectation value of the total energy $\langle \hat{H} \rangle$ for the normalized test wave function from part b. Use:

$$V_0 = \frac{\hbar^2}{ml^2}.$$

D) What does the variation principle do / what's the idea behind it?

For another normalized test wave function φ2:

$$\langle \phi_2 | \hat{H} | \phi_2 \rangle = \frac{1483}{256} \frac{\hbar^2}{ml^2}$$

Which of the two functions φ1 or φ2 represents in the sense of variational principle a better test wave function for the system?