1) First law of thermodynamics:

$$dU = \delta Q - \delta A = \underline{TdS - pdV}$$

2) Change of variables from $U(S,V)$ to $U(V,T)$:

$$F \equiv U - TS;$$

$$dF = dU - TdS - SdT = \cancel{TdS} - pdV - \cancel{TdS} - SdT =$$

$$= -pdV - SdT \quad (F(V,T) - \text{Helmholtz function})$$

3) Total differential F:

$$dF = \left(\frac{\partial F}{\partial V}\right)_T dV + \left(\frac{\partial F}{\partial T}\right)_V dT \quad (\text{defenition})$$

$$dF = -pdV - SdT \qquad (\text{see p.2})$$

and consequently:

$$\left(\frac{\partial F}{\partial V}\right)_T = -p \quad \text{and} \quad \left(\frac{\partial F}{\partial T}\right)_V = -S$$

4) Maxwell's relation:

$$\frac{\partial^2 F}{\partial V \partial T} = \frac{\partial^2 F}{\partial T \partial V} \quad (\text{truly for mixed derivative})$$

$$\frac{\partial}{\partial T}(-p)_V = \frac{\partial}{\partial V}(-S)_T ; \left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

5) $\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T ; \left(\frac{\partial^2 p}{\partial T^2}\right)_V = \frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V}$

How to get proof that $\frac{\partial S}{\partial V \partial T} = \frac{1}{T}\left(\frac{\partial C_V}{\partial V}\right)_T$?

6) So long as $\delta Q = l\,dV + C_V dT$; $\frac{\delta Q}{T} \equiv dS = \frac{l}{T} dV + \frac{C_V}{T} dT$

and for $S = S(V,T)$ $dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT$:

$$\frac{C_V}{T} = \left(\frac{\partial S}{\partial T}\right)_V \quad \left(\text{and of course } \frac{l}{T} = \left(\frac{\partial S}{\partial V}\right)_T\right)$$

$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$ and finally: $\left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial}{\partial V}\left[T\left(\frac{\partial S}{\partial T}\right)_V\right]_T = T\frac{\partial^2 S}{\partial T \partial V}$

see p.5 and enjoy: $\left(\frac{\partial^2 p}{\partial T^2}\right)_V = \frac{1}{T}\left(\frac{\partial C_V}{\partial V}\right)_T \Rightarrow T\left(\frac{\partial^2 p}{\partial T^2}\right)_V = \left(\frac{\partial C_V}{\partial V}\right)_T$