CHEE 3363 – Fluid Mechanics – Fall 2012
Special Project

Background

The chief engineer of your company asks you to design a cylindrical vortex separator for separating oil from oil-water mixtures arising in your production operations. The oil is entrained in water as a distribution of fine droplets, with a range of droplet sizes from a few microns to hundreds of microns, and oil concentrations by weight being typically less than 20%. Mixtures with droplet sizes and concentrations above these ranges are handled with other separation processes (gravity settling or other methods); vortex separation is best for fine droplet mixtures.

As illustrated in Figures 1 and 2, the separator is relatively simple with an axial fed cylindrical housing with a cylindrical frit, the latter consisting of a pipe with small holes in the walls. To achieve centrifugal driven separation of the oil from the water, the outer housing, frit, and drive shaft rotate as a unit at a steady angular speed ω (Figure 2). Because the oil is lighter than the water, the oil droplets are driven to the frit where they enter the frit region, coalesce, and exit as an oil stream. The oil-water mixture enters from the left and exits from the right and the separated oil exits from the frit exit line. Once in the frit line, the oil does not re-enter the water concentrated chamber.

As an approximation, in the z-direction, the oil drops are carried along with the water stream at the same velocity as the stream, with the mixture velocity and volumetric flow rates being that of steady axial annular flow between the housing and the frit. The velocity, average velocity, and flow rate at any position z are describe by:

$$v_z = \frac{KR_H^2}{4\mu}\left[1-\left(\frac{r}{R_H}\right)^2 - \frac{\left(1-\kappa^2\right)}{\ln(\kappa)}\ln\left(\frac{r}{R_H}\right)\right] \qquad (1)$$

$$K = \frac{p_0 - p_L}{L} = \frac{\Delta p}{L} \qquad \kappa \equiv \frac{R_F}{R_H} \qquad (2a,b)$$

and
$$\bar{v}_z = \frac{KR_H^2}{8\mu}\left[1+\kappa^2+\frac{\left(1-\kappa^2\right)}{\ln\kappa}\right] \tag{3}$$

$$Q = \frac{\pi KR_H^2}{8\mu}\left[\left(1-\kappa^4\right)+\frac{\left(1-\kappa^2\right)^2}{\ln\kappa}\right] \tag{4}$$

where R_H and R_F are the radii of the housing and frit, K the pressure drop driving force with L being the length of the rotating housing and frit section, and μ the viscosity of the continuous water phase. Similar equations are applicable for flow in the frit-shaft annulus, but with $\kappa = R_S/R_F$.

In the θ-direction, the flow is rigid body rotation (neglecting end effects),

$$v_\theta = \omega r \tag{5}$$

and in the r-direction the droplet velocity toward the frit (relative to the water stream) is given by:

$$\hat{v}_r = \frac{dr}{dt} = \frac{1}{18}\left(1-\frac{\rho_{oil}}{\rho}\right)\frac{d^2\omega^2 r}{\nu} \tag{6}$$

where r is the radial position of the drop, t the time, and ρ's are the the densities of the water and oil, d the drop diameter, and $\nu = \mu/\rho$, the kinematic viscosity of the water. Equation (6) is a result of the balance of the net centrifugal force on the drop and the viscous drag force of the water resisting the motion ($F_D = 6\pi v_d \mu d$, which is Stokes law).

In order for the separator to function properly, the flow must be laminar in the annular regions. Turbulence will produce dispersive mixing which will negatively impact the separation and must be avoided. To determine the critical Reynolds number for flow in the annulus, it is suggested that you use the hydraulic diameter approach (page 368 in your text) to estimate the critical Reynolds number for flow in an annulus. The critical Reynolds for a tube is used except the tube diameter is replaced by the hydraulic diameter.

Project objectives:

- Subject to the constraints of laminar flow, determine the performance measures (the separation percent, the volume rate of oil removed per unit time, and possibly others; you select) the geometrical parameters (R_H, R_F, and L), the operational parameters (ω and Q), for one pass separation of oil-water mixtures with different oil-water concentrations by weight (< 30% of oil) as well as drop size distributions characterized by d_{max} and d_{min} with the cumulative distribution given by

$$F = \sin\left(\frac{\pi(d - d_{\min})}{2(d_{\max} - d_{\min})}\right)$$

  where $d_{min} >= 10$ microns and $d_{max} <= 1000$ microns. As a base case, start with a 15% oil-water mixture with d_{min} = 50 microns and d_{max} = 300 microns and R_H = 0.08 m, R_F = 0.015 m, and R_S = 0. 008 m; then explore other concentrations and distributions around this base case.

- Based on the above, suggest specific geometric parameters and operating conditions most suitable for the one pass removal of different distributions and oil-water concentrations.

- Finally suggest other geometric configurations that might be more effective than the simple cylindrical rotation housing system used here.

Fluid data

For water ρ = 1000 kg/m3, ν = 1E-6 m2/s; for oil, use ρ = 900 kg/m3, , ν = 1E-4 m2/s. Other data which might be needed will be provided on request.

Project Report (follow these instructions carefully)

Your results are to be given in a formal report (10 pages max in report body with additional pages in the Appendix). In the body, give a brief background (1/2 page), then summarize the key issues, the needed equations, your approach, your results, and your conclusions. Use carefully thought out tables, figures, equations that support your results and conclusions. Put equation development/derivations, detailed analysis, spreadsheet results in the Appendix. Also, if you reference an equation, table, or figure in the body, it must appear in the body and not just the Appendix. NOTE: You should derive all equations used in your analysis and these developments should appear in a special section of the Appendix; don't trust any equations given here (or elsewhere); there could be errors; you need to check. ***The success of your work will be judged not only on the quality and creativity of the technical work, but also the quality of your written presentation, interpretation, analysis, and conclusions.*** A lengthy, sloppy report, with a poor presentation of the results will not be useful and your performance evaluation will be poor. Keep it simple, direct, neat, and easy to understand.

Project due date: Monday, December 10, 12 noon.
Put in my departmental mail box.

Figure 1: Oil-Water Separator

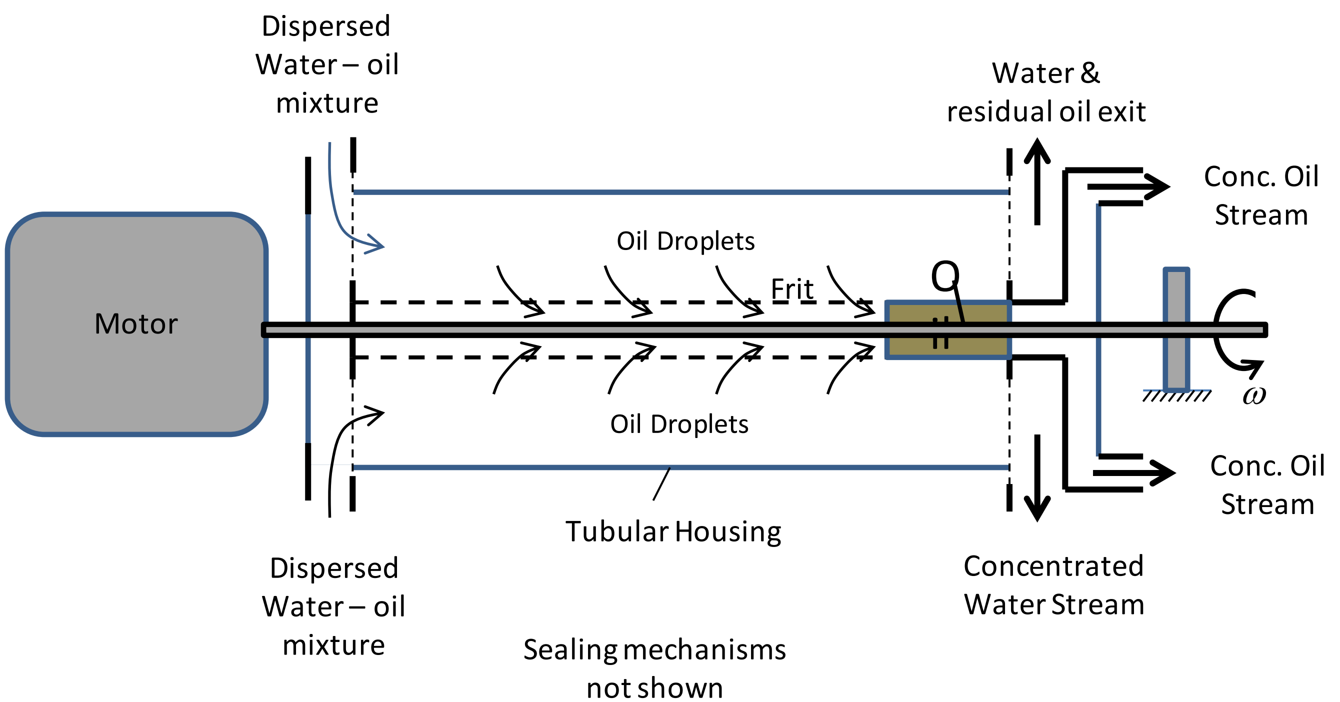
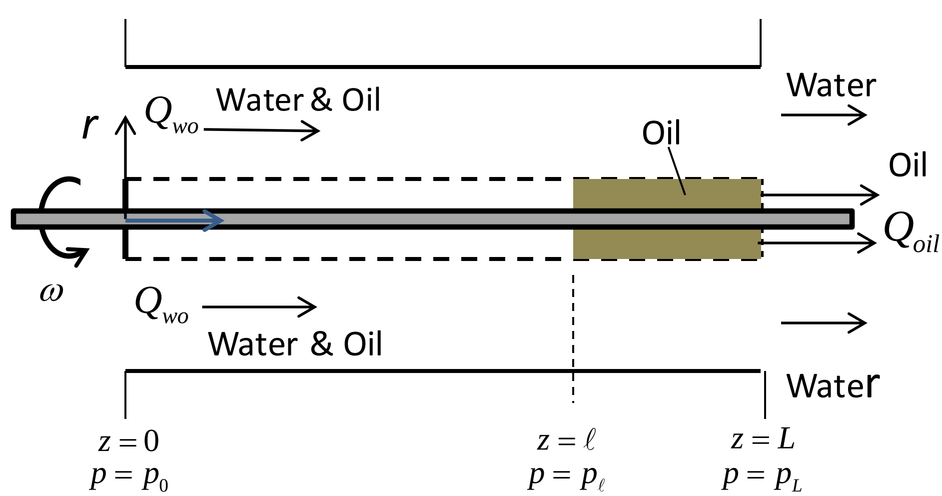


Figure 2. Rotating Housing, Frit, and Shaft Section



Housing radius: R_H
Frit radius: R_F
Shaft radius: R_S